# NAVAL POSTGRADUATE SCHOOL
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# THESIS

THE DECOMPOSITION
OF AN ARBITRARY THREE-DIMENSIONAL
PLANAR POLYGON INTO A
SET OF CONVEX POLYGONS

by

Jeffrey H. Potts

December 1986

Thesis Advisor: Michael J. Zyda

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# The Decomposition Of An Arbitrary Three–Dimensional Planar Polygon Into A Set Of Convex Polygons

by

**Jeffrey Hal Potts**
Captain, United States Marine Corps
B. S.,United States Naval Academy, 1977

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requirements for the degree of

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## ABSTRACT

We present in this study a three step algorithm for the decomposition of arbitrary, three-dimensional, planar polygons into convex polygons. Through a series of translations and rotations, an arbitrary polygon is mapped onto the x-y plane, then broken into a set of convex polygons, and finally mapped back to the polygon's original coordinate system for filling and display by special graphics hardware.

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# I. INTRODUCTION

Computer hardware advancements applied to the field of computer graphics have recently made possible the real-time display and animation of three-dimensional object models. In most currently produced graphics systems, the three-dimensional (3D) object models are comprised of sets of 3D polygons. For years the key bottleneck to generating real-time images of 3D models has been the limited speed with which the 3D polygons comprising the model have been filled. This bottleneck has been alleviated recently with the advent of special hardware designed to fill polygons at rates exceeding 44 million pixels per second [Ref. 1]. For real-time animation of 3D object models, such polygon fill rates are essential.

The majority of the graphics systems that provide polygon fill hardware only provide hardware for filling convex polygons. Concave polygons passed to such hardware usually are filled incorrectly, if at all. To utilize the capabilities of such hardware for the general polygon case, that is, a mix of concave and convex polygons, it is necessary to have some means of decomposing an arbitrary 3D planar polygon into a set of 3D planar convex polygons completely covering the area defined by the original polygon. The following study attempts to provide such an algorithm. We then discuss the implementation and performance characteristics of that algorithm on the Silicon Graphics, Inc. IRIS workstation.

# II. BACKGROUND

## A. DEFINITIONS

A polygon is defined as a geometric figure that is described by an array of x, y, and z coordinates for its vertices. A two dimensional polygon lying in the x-y plane is a special case of a three dimensional polygon in which the z coordinates are all zero. A convex polygon is one that has no points of negative curvature along its boundary [Ref. 2: p. 217]. This means that a line can be drawn from any point inside the polygon to any of its vertices and have the line lie entirely within the polygon boundary [Ref. 3: p. 350]. A concave polygon fails the above test, i.e. we cannot draw a line from every point inside the polygon to every vertex and have the line remain within the polygon boundary. However, any point inside a concave polygon can be shown to lie inside a convex polygon created by some of the vertices and sides of the original polygon [Ref. 3: p. 330]. Figure 2.1 provides examples of convex and concave polygons.

## B. DECOMPOSITION

To fill a concave polygon utilizing hardware that only correctly fills convex polygons, we must first break down the polygon into simpler convex polygons that together cover the area of the original. The process of decomposing a concave polygon does not necessarily yield a single unique set of convex polygons, but the

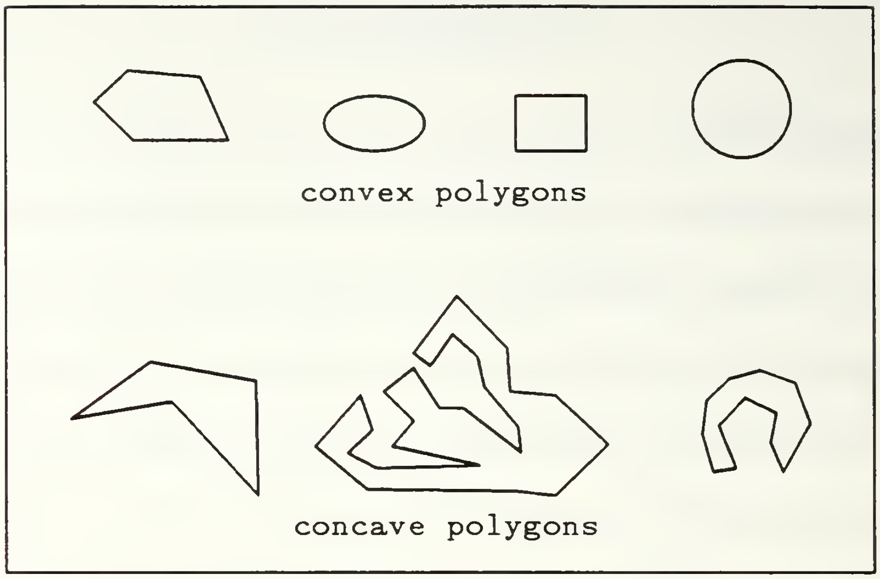


Figure 2.1 - Convex and Concave Polygons

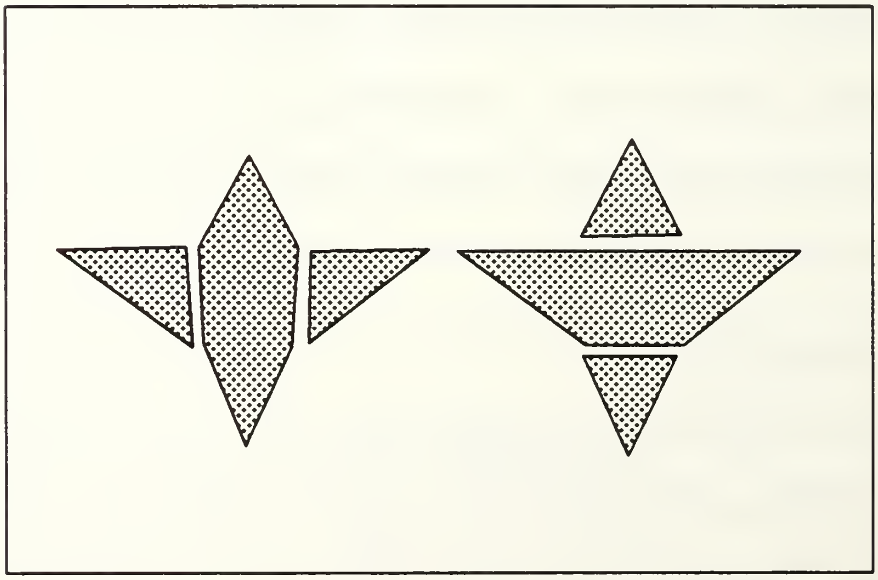


Figure 2.2 - Two Equivalent Decompositions of a Concave Polygon.

result of two different decompositions of the same polygon should be equivalent as shown in Figure 2.2.

During the operation of decomposing a polygon, there are two main checks that must be conducted to ensure complete and proper decomposition takes place. The first check is to ensure that no area of a created convex polygon falls outside the boundary of the original concave polygon. The second check is to ensure that the concave polygon is completely covered by the total areas of the created convex polygons. Figure 2.3 provides examples of the two checks.

The polygon decomposition algorithm produced for this study is comprised of three basic steps. The first maps the three dimensional coordinates of the polygon into a two dimensional plane. This process generates a polygon that is more easily examined and decomposed. The second step conducts the actual decomposition an performs the two decomposition checks. The third step maps the coordinates of the created convex polygons back to the the original orientation in three-space. At this point, the created convex polygons are then ready for passing to the graphics system's polygon fill hardware.

## C. RESEARCH FACILITIES

The IRIS (Integrated Raster Imaging System) 2400 Graphics Workstation, manufactured by Silicon Graphics, Inc., is the target system for our polygon decomposition algorithm. By incorporating custom VLSI chips to replace slower graphics software, the IRIS system can accept polygons in user-defined

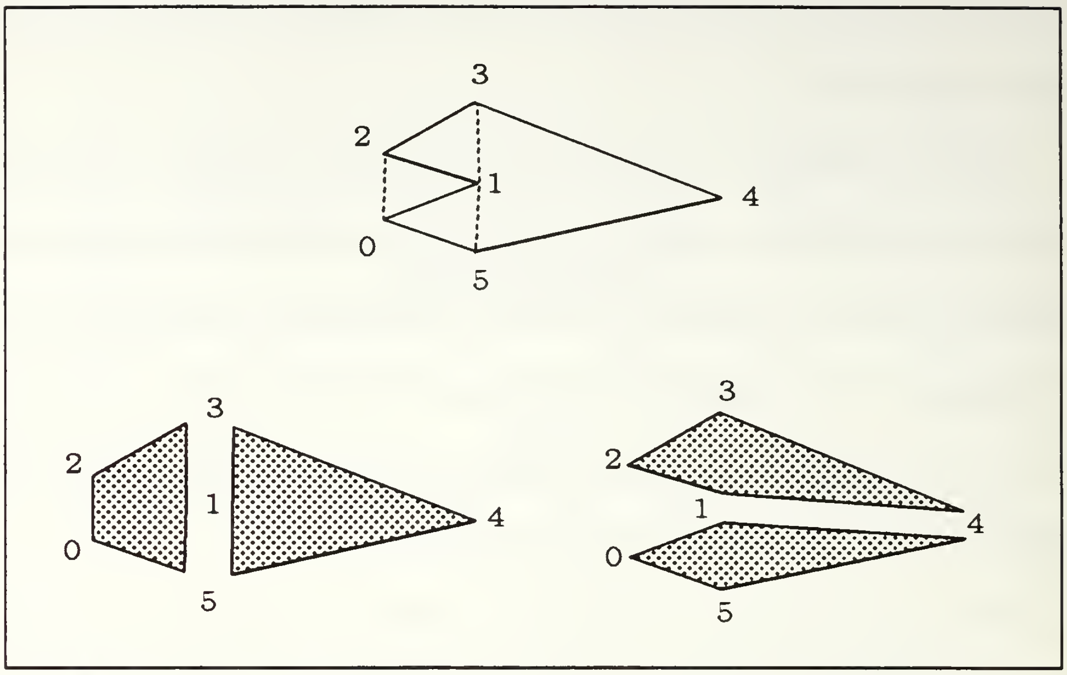


Figure 2.3a - Without a check to ensure the convex polygon does not fall outside the bounds of the original polygon, vertices could be chosen that yield an incorrect decomposition.

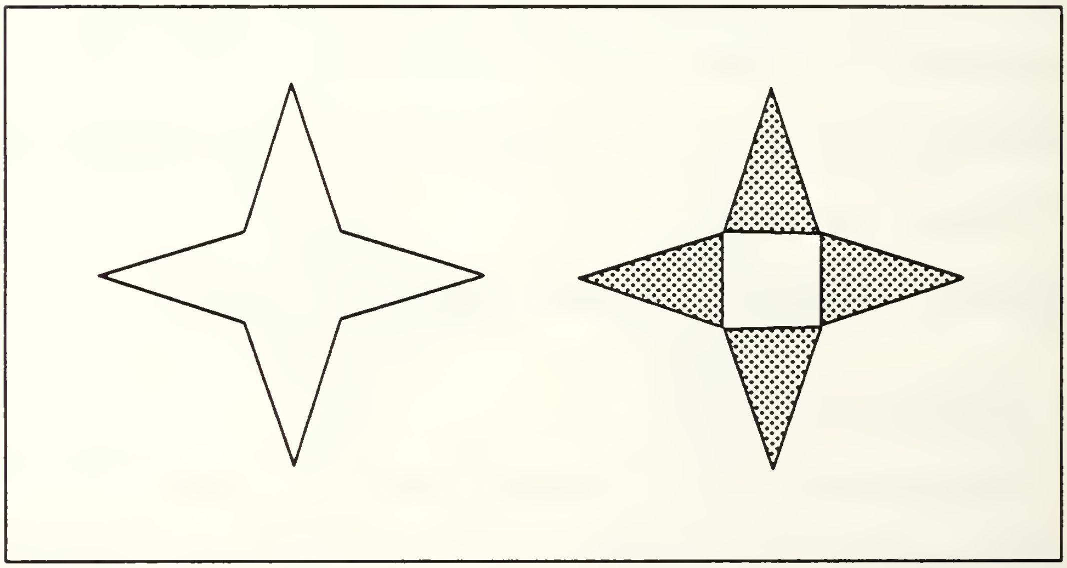


Figure 2.3b - The second decomposition check ensures the concave polygon is completely covered by the decomposed polygons. In this case all vertices and edges were utilized, however a hole is still left.

coordinates, and transform them to screen coordinates for display on a color graphics monitor that provides 1024 x 768 resolution. This use of special hardware yields processing speeds that are some 100 times faster than similar operations carried out in software.

# III. STEP I: MAPPING THE 3D POLYGON TO THE 2D X-Y PLANE

## A. THE NORMAL TO A POLYGON

The first step of the polygon decomposition algorithm, is comprised of several small steps. The goal of these steps is to map the polygon to the x-y plane. First, three non-colinear points are selected from the array of x, y, and z coordinates of the polygon. These are used to define the equation of the plane for the 3D polygon, and the normal to that plane (Figure 3.1).

$$\mathbf{Ax + By + Cz = D}$$

$$\vec{\mathbf{N}} = \mathbf{A}_{\vec{\mathbf{i}}} + \mathbf{B}_{\vec{\mathbf{j}}} + \mathbf{C}_{\vec{\mathbf{k}}}$$

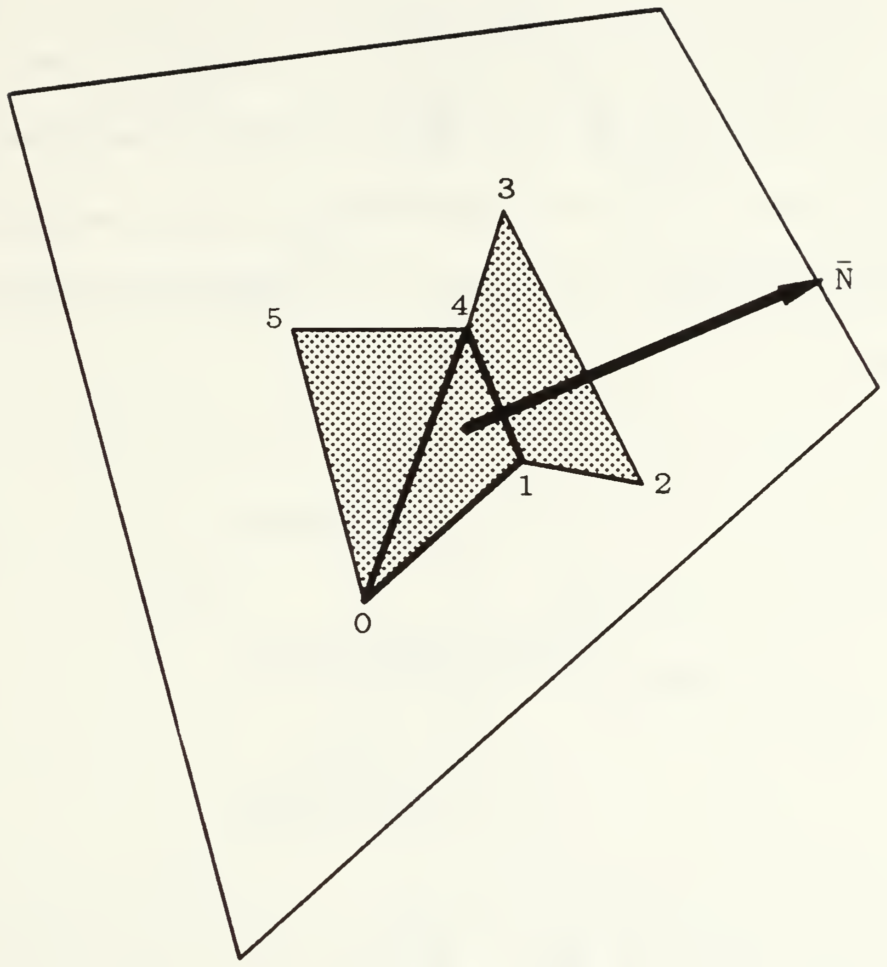
The coefficients A, B, and C are calculated from the coordinates of the three selected points by the following equations:

$$\mathbf{A = (y_0 - y_1)(z_2 - z_1) - (y_2 - y_1)(z_0 - z_1);}$$

$$\mathbf{B = (z_0 - z_1)(x_2 - x_1) - (z_2 - z_1)(x_0 - x_1);}$$

$$\mathbf{C = (x_0 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_0 - y_1).}$$

The coefficients are then used to solve for D of the equation of the plane.



Three vertices 0, 1, and 4
are selected to determine the
Normal to the plane.

Figure 3.1 The Normal to a Polygon

## B. ANGLES ABOUT THE AXES

Once the equation of the plane is determined, each point in the polygon input array is checked to ensure that the polygon is in fact planar. If the polygon passes this test, then the angles between the normal and the axes of the coordinate system are found. From these angles, the necessary rotations, about the x and y axes, to bring the polygon into the x-y plane, are calculated. The following equations [Ref. 4: pp. 24-59] are used to calculate the angles shown in Figure 3.2a.

$$\cos \alpha = \frac{A}{\sqrt{A^2 + B^2 + C^2}}$$

$$\cos \beta = \frac{B}{\sqrt{A^2 + B^2 + C^2}}$$

$$\cos \gamma = \frac{C}{\sqrt{A^2 + B^2 + C^2}}$$

For the polygon to lie in the x-y plane, the angle between the normal and the z axis is either zero or 180 degrees. Necessarily then rotations are conducted to bring the angles between the normal and the x and y axes to 90 degrees each.

First a projection of the normal onto the x-z plane is made as shown in Figure 3.2b. The dot product of that projected normal $\vec{N}'$ and the z axis is calculated. The dot product is divided by the product of the magnitudes of the two vectors yielding the cosine of the angle of rotation about the y axis ($\phi$).

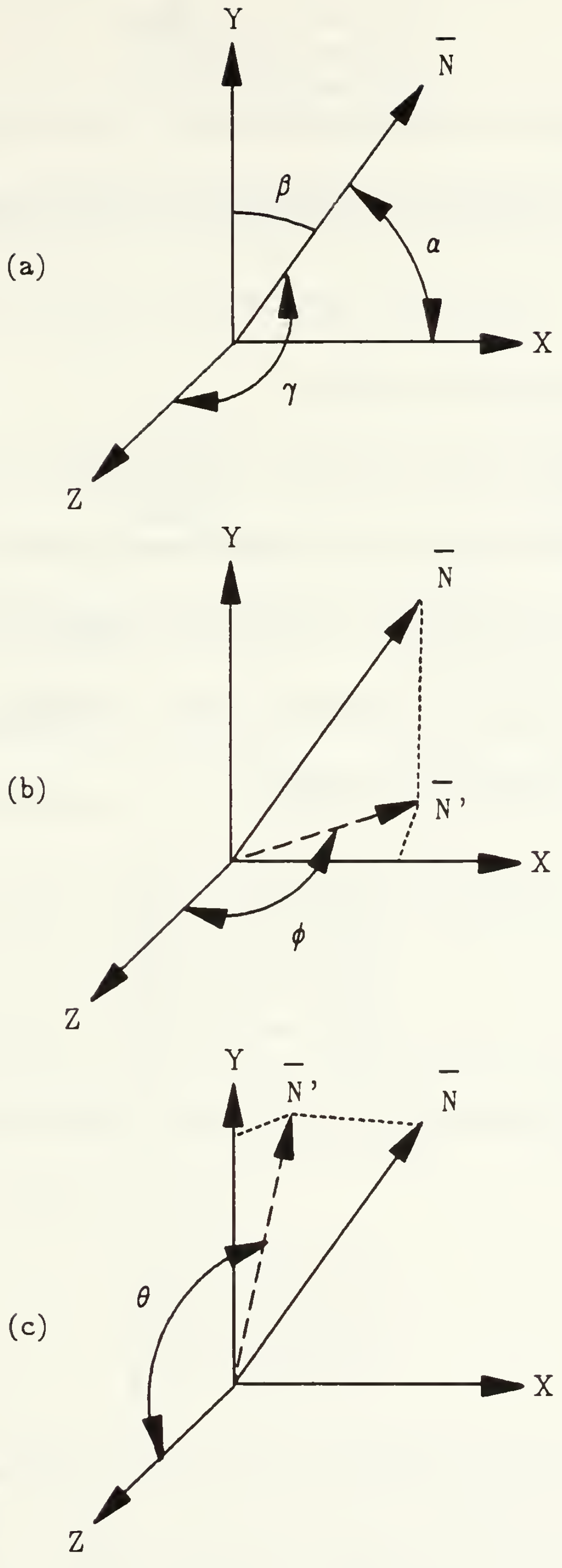


Figure 3.2 Angles about the Axes

$$\cos \phi = \vec{N}' \frac{\vec{Z}}{|\vec{N}'|\ |\vec{Z}|}$$

A second projection of the normal onto the y-z plane is made and the angle of rotation about the x axis ($\theta$) is calculated in the same manner as above (Figure 3.2c).

## C. TRANSFORMATION MATRIX

To actually rotate the polygon into the x-y plane, a transformation matrix must be created that is used in the multiplication of the polygon's coordinates. We first translate the polygon to the origin by adding the negative x, y, and z coordinates of one of the polygon's vertices. Assuming l, m, and n are the coordinates selected, the matrix becomes:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -l & -m & -n & 1 \end{bmatrix}$$

A rotation matrix for the angle $\phi$ about the y axis is then created

$$\begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and concatenated with the translation matrix. This is followed by a multiplication with a rotation matrix for the angle $\theta$ about the x axis.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The result of these matrix concatenations is the following transformation matrix that is used to map the array of vertices onto the x-y plane.

$$\begin{bmatrix} \cos\phi & -\sin\phi\sin\theta & -\sin\phi\cos\theta & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ \sin\phi & -\cos\phi\sin\theta & \cos\phi\cos\theta & 0 \\ -l\cos\phi - n\cos\phi & -m\cos\theta + (l\sin\phi - n\cos\phi)(-\sin\theta) & -m\sin\theta + (l\sin\phi - n\cos\phi)(\cos\theta) & 1 \end{bmatrix}$$

Once we have this matrix set up. call it $M_{toplane}$, we then use it to transform the polygon's coordinates via a simple row vector by matrix multiplication. The result of this multiplication is a transformed set of coordinates. in which the z coordinates are zero. The x and y values of this transformed array are then used in the decomposition step of our algorithm. We note at this point that for the mapping of the coordinates back to the original vertices (Step III of the Alg.), that the index into the transformed array is also the index for the untransformed point in the original array of vertices.

# IV. STEP II: POLYGON DECOMPOSITION

The decomposition step divides the polygon into multiple convex polygons. In so doing. the algorithm performs the two checks to ensure that no area outside the polygon is shaded and that the created convex polygons completely cover the area of the concave polygon. This step requires that the vertices of the polygon be ordered in a clockwise fashion. with respect to the +z directed normal to the x-y plane. Thus a determination of the ordering must be made and if necessary the array of vertices reversed.

## A. CLOCKWISE ORDERING

To determine if the vertices of the polygon are ordered clockwise. the polygon's area is calculated using the formula [Ref. 5: p. 369] below. (Note $x_i, y_i$ are the polygon's 2D coordinates.)

$$\frac{1}{2}((x_0y_1)+(x_1y_2)+\cdots+(x_{n-1}y_n)+(x_ny_0)-(y_0x_1)-(y_1x_2)-\cdots-(y_{n-1}x_n)-(y_nx_0))$$

The above equation yields a negative area for clockwise and a positive area for counterclockwise polygons. If the polygon is ordered counterclockwise then the array of x, y, and z coordinates that define the polygon is reversed before the selection procedure is begun.

## B. VERTEX EXAMINATION

The selection of vertices for a convex polygon is conducted by examining each vertex of the concave polygon with respect to the previously selected vertices of the developing convex polygon. This is begun by initially selecting two adjacent vertices, then in clockwise order, examining each of the polygon's remaining vertices to build as large a convex polygon as will fit within the boundary of the concave polygon.

A series of tests are conducted in the selection of vertices for the convex polygon. As vertices are selected, the line segments connecting these become the edges of the convex polygon and define its boundary.

### 1. Test One

The first test of a vertex is if it lies to the left or right of each line segment of the developing convex polygon. The vertices of the concave polygon are examined in clockwise order. Thus if a vertex lies to the left of any of the line segments of the developing polygon, then a concave angle is necessary to include the vertex in the polygon. As shown in Figure 4.1a, vertex 2 lies to the left of the line segment 0-1. A concave angle ( $\rho$ in Figure 4.1b) is necessary to add vertex 2 to the developing polygon. Therefore vertex 2 is not selected and the process moves on to perform the same test on vertex 3.

### 2. Test Two

A vertex that passes the first test may still require a concave angle to be included in the developing convex polygon. In Figure 4.2, the vertices 0, 1, 2, 3,

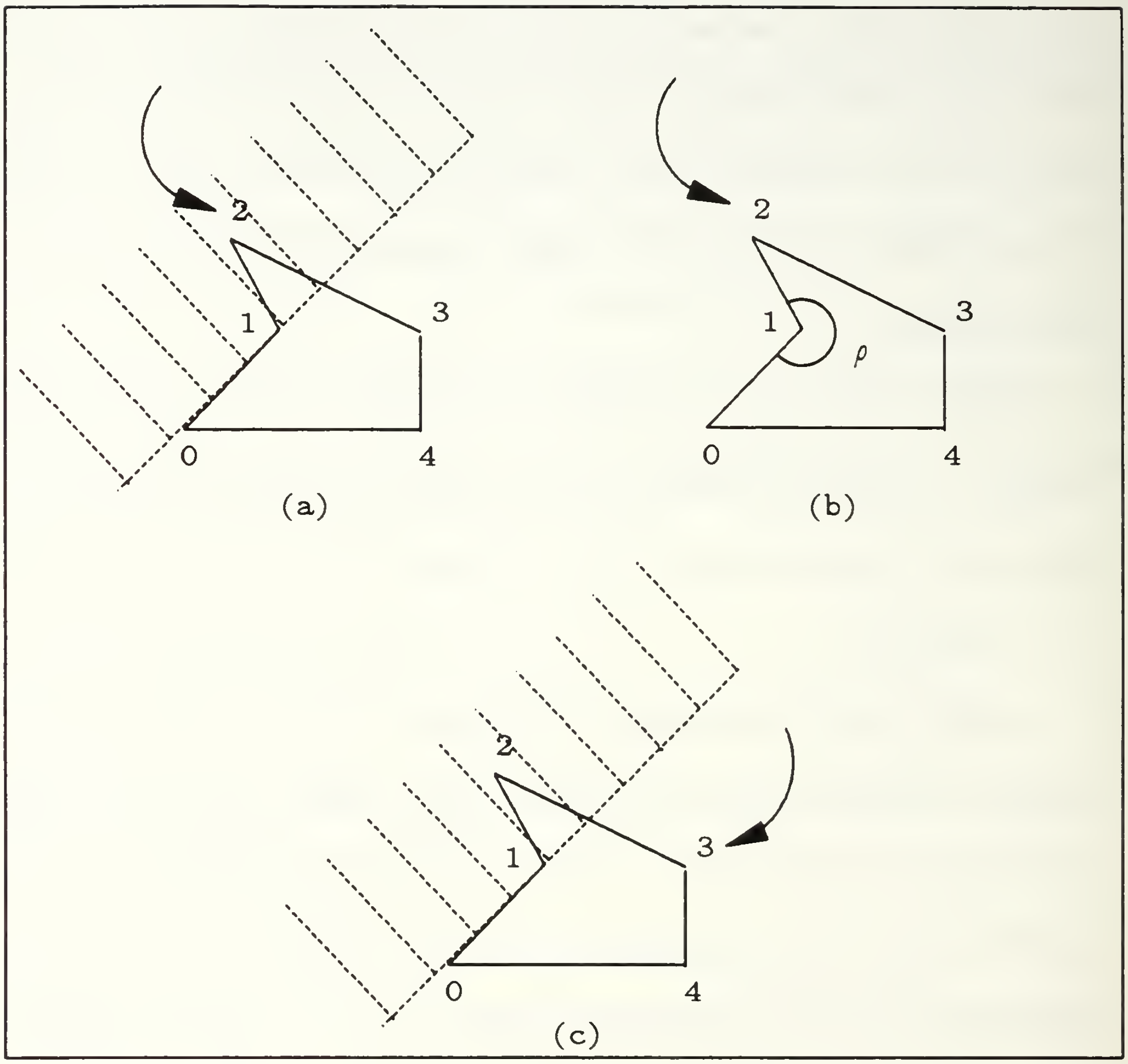


Figure 4.1 - (a) Vertex 2 lies to the left of the line segment 0-1, (b) Concave angle ($\rho$) would be required to connect Vertex 2 to the developing polygon, (c) Vertex 2 is skipped and the examination is continued with vertex 3.

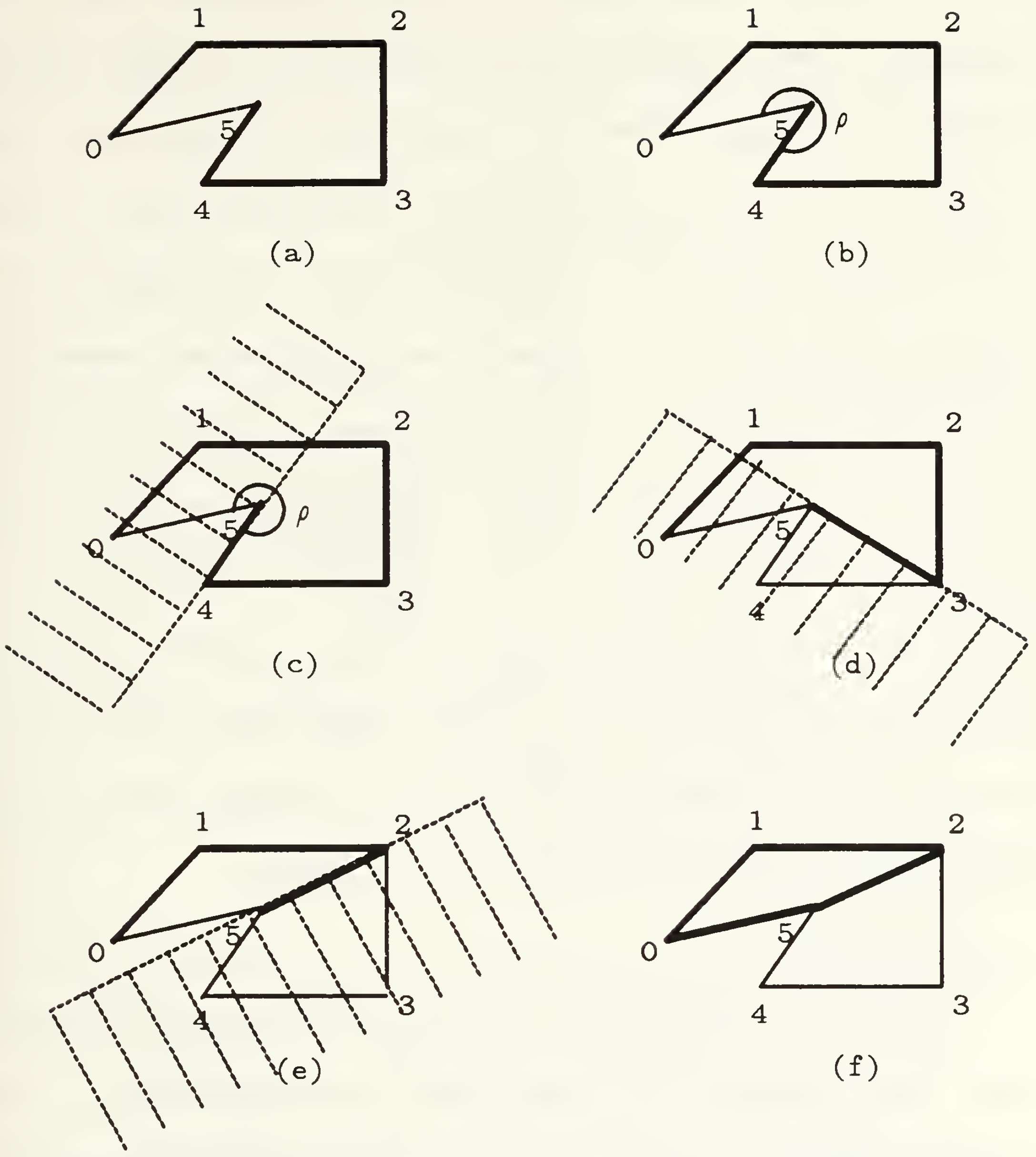


Figure 4.2 - (a) Vertex 5 lies to the right of each line segment of the developing polygon, (b) Concave angle ($\rho$) is required to connect Vertex 5 to Vertex 0, (c) Concave angle ($\rho$) can be identified since 0 lies to the left of line segment 4-5, (d) Vertex 4 is skipped and the test repeated with line segment 3-5, (e) Vertex 3 is skipped and the test repeated with line segment 2-5, (f) Result of the tests are that vertices 0, 1, 2 and 5 are selected.

and 4 have been previously selected for a convex polygon with vertex 5 under examination. Vertex 5 passes the first test since it lies to the right, or inside of each of the line segments. However, a concave angle ($\rho$ in Figure 4.2b) is again required to connect vertex 5 with vertex 0 of the developing polygon. A concave angle of this type is identified, as shown in Figure 4.2c, if the initial vertex of the developing convex polygon lies to the left of the line segment created by the vertex under examination and the last selected vertex of the developing polygon. In Figure 4.2c, the initial vertex, 0, lies to the left of line segment 4-5. To remedy this, the last selected vertex of the developing convex polygon, vertex 4, is removed and the test is again conducted with the line segment formed by the vertex under examination and the last selected vertex (Figure 4.2d & 4.2e). This process is repeated, removing vertices from the developing polygon, until the initial vertex no longer lies to the left of a new line segment.

### 3. Test Three

The third test conducted is to ensure that no line segment created in the convex polygon intersects any of the edges of the concave polygon. Such an occurrence results in area outside the concave polygon being included in the convex polygon and an inaccurate polygon fill taking place. In Figure 4.3, the previously selected vertices are 0, 1, 2, and 3. Vertex 7 is under examination and passes the two earlier tests. However, the selection of vertex 7 results in the convex polygon 0, 1, 2, 3, 7 which contains area outside the concave polygon. A situation such as this is identified since line segment 3-7 intersects line segment

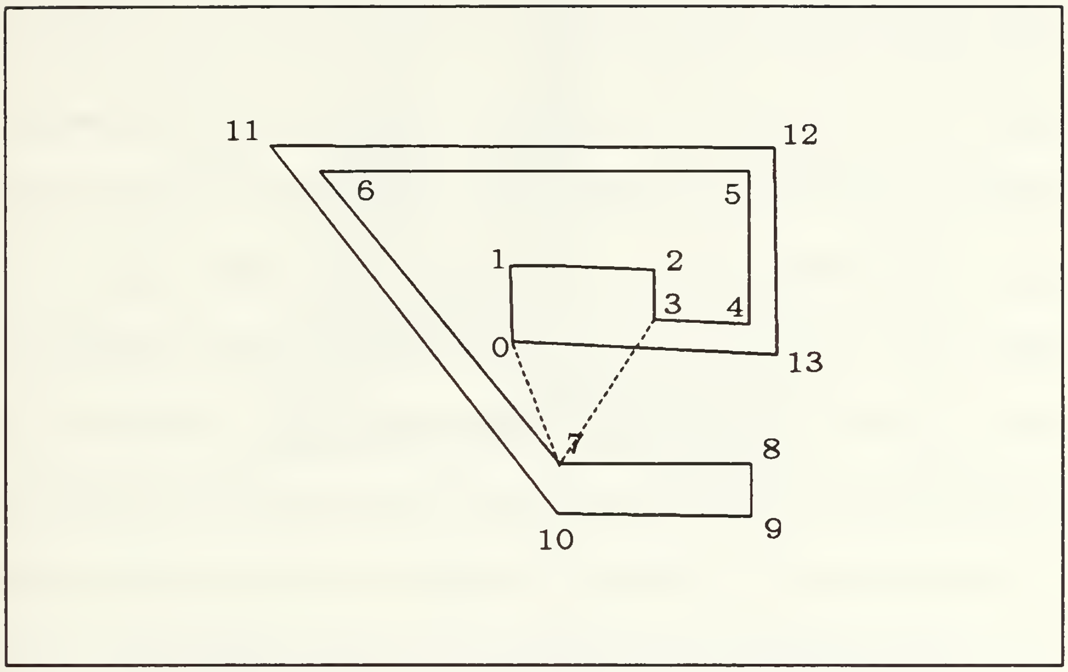


Figure 4.3 - Tests must be conducted for the intersection of developed line segments with the edges of the original concave polygon. Otherwise Vertex 7 would be an acceptable vertex for inclusion in the developing polygon 0, 1, 2, 3...

13-0 of the concave polygon. When this occurs, the vertex under examination is removed from the convex polygon and the next vertex of the concave polygon is examined.

## C. SELECTION COMPLETION

The selection process for the developing convex polygon concludes when all vertices of the concave polygon have been examined. The convex polygon is then ready for the third step of our algorithm, mapping to the original polygon's 3D coordinates. Figure 4.4 is an example of a concave polygon that was decomposed to the three shaded convex polygons using the above three tests. We have a "hole" left in the concave polygon even though all vertices and edges of the concave polygon have been utilized. Holes such as this are prevented by taking the initial and last vertices of a developed convex polygon as the starting vertices of a convex polygon to be developed. In Figure 4.4, vertices 0 and 2 of polygon 0, 1, 2 are taken to develop convex polygon 0, 2, 4 to fill in the "hole". Once this is accomplished, step two of the algorithm is repeated by selecting a vertex of the concave polygon that has not yet been utilized in a convex polygon. The decomposition process is complete when there are no vertices or edges of the concave polygon that have not been utilized in one or more convex polygons.

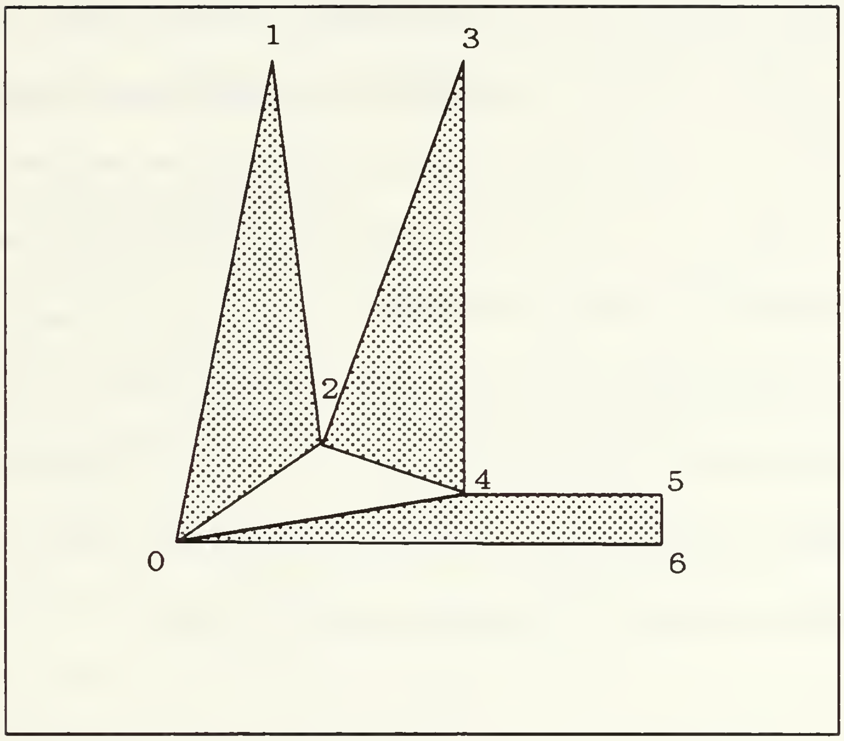


Figure 4.4 - All vertices and edges of the polygon have been used, yet a hole remains.

# V. STEP III: MAPPING TO THE ORIGINAL COORDINATE SYSTEM

The second and third steps of the decomposition process are interleaved, i.e., each created convex polygon is immediately mapped back to the original coordinate system. This is accomplished by maintaining the original array of vertices that the user provided for the three-dimensional polygon. As a convex polygon is produced, it is actually represented by the indices of its vertices. Actual vertex coordinates are extracted from the vertex array, as required, using the index. In this manner, the transformed coordinates can be utilized during decomposition and the index can be used to obtain the original coordinates for final display. By this method, there is no consequential need to translate and rotate the vertices back to the original coordinate system.

# VI. IRIS IMPLEMENTATION

Implementation of the decomposition algorithm on the IRIS system is via approximately 1600 lines of C code (Appendix). Several data structures and various IRIS system hardware features are utilized as described below.

## A. MATRIX IMPLEMENTATION

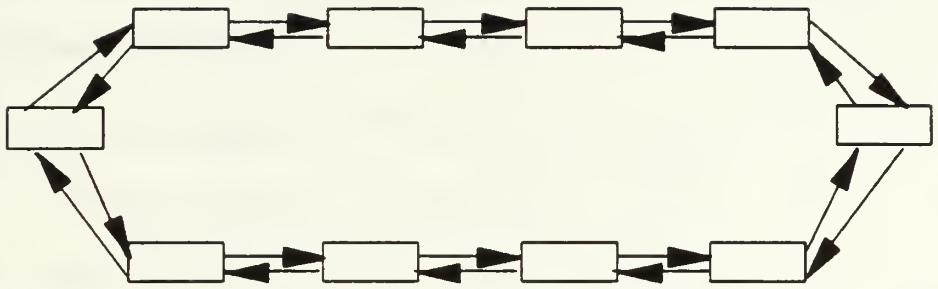
In the development of the transformation matrix, IRIS system calls for translation and rotation are used to alter the top of the system matrix stack. These routines perform the matrix concatenations needed to translate and rotate the polygon to the x-y plane. To convert the polygon vertices to 2D coordinates, four vertices at a time are loaded into a matrix. The x, y, and z coordinates go into the first three columns with 1's filling the fourth column to provide the necessary homogeneous coordinates. This is placed on the system's matrix stack and multiplied with the transformation matrix using the IRIS's Geometry Engines. The transformed coordinates are removed from the matrix stack and the process is repeated until all vertices have been transformed.

## B. DOUBLY LINKED LIST

A data structure for each vertex of the polygon is maintained in our system. This structure contains the vertex's index into the vertex array. As actual coordinates are needed, the vertex's index is used to extract the x, y, and z coordinates from this array. A "used" flag indicates the vertex's use in a developed convex polygon, with a slope and y-intercept stored for the line segment the vertex forms with its predecessor in the array. A forward pointer connects the structure with the next vertex in clockwise order, while a back pointer connects the structure with its preceding vertex. This produces a doubly linked circular list of vertex structures that permits forward clockwise traversal of the polygon and rearward counterclockwise traversal (Figure 6.1).

## C. VERTICES STACK

To manage the examination and selection of vertices for a developing convex polygon, a stack is utilized. The size of the vertices stack is determined at run time when contiguous memory is dynamically allocated, based on the number of vertices in the concave polygon. As a vertex is examined, its index is placed on top of the stack along with the slope and y-intercept of the line segment it creates with the vertex below it. If the vertex passes all selection tests, it remains on the stack and a new vertex is pushed on top. On the other hand, when a vertex fails a selection test, it is popped off the stack and the next vertex to be examined is placed on top. Once the selection of vertices for a convex polygon is completed, the vertices are passed to the IRIS system hardware and the stack is cleared to continue with the decomposition process.



(a) Doubly linked circular list of structures,

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| Rearward Pointer | Index | Used Flag | Slope | Y-intercept | Forward Pointer |

(b) Vertex structure

Figure 6.1

# VII. LIMITATIONS AND CONCLUSIONS

This study has presented an algorithm for the decomposition of an arbitrary, planar polygon into a set of convex polygons, thus eliminating the need to manually perform such decomposition. With the algorithm's dynamic allocation of memory, the size polygon that can be filled is limited only by the amount of memory available for use. The algorithm indirectly provides an extended capability over the IRIS system's limit of 384 vertices in one polygon (Ref. 6: p. 2).

However due to the time required to test each vertex for concavity, the algorithm is not suggested for use where real-time response is important. To demonstrate the overhead involved in testing for concavity, a convex polygon was filled and displayed 500 times by the algorithm. The same polygon was then filled utilizing only the IRIS's polygon fill hardware. The results are provided in Figure 7.1 for various polygon sizes.

Due to the three primary steps necessary for decomposition and the tests conducted on each vertex, the speed of the algorithm depends entirely upon the number of vertices and concave angles of the polygon. As such, the success of the algorithm's use in real-time display and animation is based upon a balance between the complexity of the concave polygons used and the speed requirements. We suggest that the created concave polygons be computed once and saved.

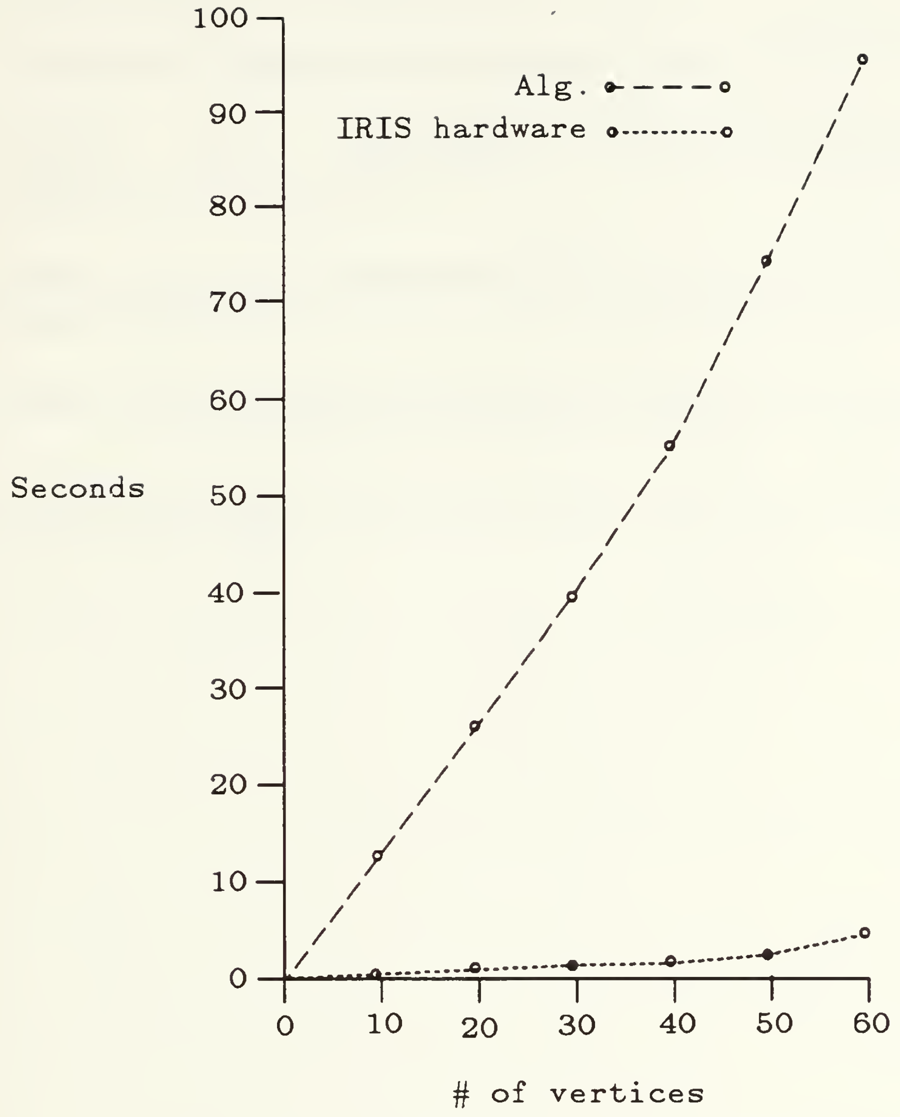


Figure 7.1 - Time required to fill and display 500 convex polygons with the number of vertices indicated

For IRIS usage, this is perhaps best accomplished through the use of the "object" mechanism of the IRIS graphics library.

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# APPENDIX – ALGORITHM SOURCE CODE

```
/**************************************************************
              globals.h
**************************************************************/

# include <gl.h>
# include <device.h>
# include <stdio.h>
# include <math.h>

/**************************************************************
               GLOBAL CONSTANTS
**************************************************************/

# define NULL 0
# define BOTTOM 0
# define  new(x) (x *) malloc(sizeof(x))

/**************************************************************
                GLOBAL TYPES
**************************************************************/

typedef int boolean;

/* Define a structure to contain data about a vertex of the polygon.
   These structures will be used to create a linked list. */

typedef struct point {
  struct  point *back;
  int     index;
  float   slope;
  Coord   intercept;
  int     direction;
  boolean failed;
  boolean segment_used;
  struct  point *forward;
};

typedef struct point point;
```

```
/* Define a structure for storing data about each vertex.  This will
   be used to create a stack for later use. */

typedef struct stack {
  int      index;
  float    slope;
  Coord    intercept:
  int      direction;
};

typedef struct stack Stack;

/****************************************************************
           GLOBAL DATA STRUCTURES AND VARIABLES
****************************************************************/

Stack  *stack;

point  *start_ptr. *current_ptr;

int    TOP;

boolean clockwise_error_flag;
```

```
/****************************************************************
  ROUTINE: convert.c
****************************************************************/
  This routine takes th: initial array of vertices, passed by
  the user, and makes the calls to other routines to perform the
  necessary translation and rotations to transform the array into
  the x-y-0 plane This new array is then used by the decomoser to
  breakdown the polygon into convex polygons.
****************************************************************/

cpolf(total_pts, pts)

int total_pts;
Coord pts[][3];

{
  int pt1, pt2, pt3, cnt;
  float A, B, C, D, anglex, angley, anglez, *new_pts;
  Matrix transform1;
  Matrix points;

  stack=(Stack *) calloc(total_pts + 1, sizeof(Stack));

/****************************************************************
  dynamically allocate memory to create an array to hold the
  vertices of the newly transformed polygon.
****************************************************************/

   new_pts= (float *) calloc (3*total_pts, sizeof(float));

/****************************************************************
  get 3 non-colinear vertices from the polygon to be used for
  calculating the normal to the polygon.
****************************************************************/

  extract_3_pts(total_pts, pts, &pt1, &pt2, &pt3);

/****************************************************************
  calculate the coefficients for the equations to the plane
  of the polygon and the normal to that plane.
****************************************************************/

  determine_coeffs(total_pts, pts, pt1, pt2, pt3, &A, &B, &C, &D);
```

```
/*************************************************************
 check to ensure all vertices of the polygon lie in the same
 plane. If so then calculate the necessary angles of rotation
*************************************************************/

 if(planar_polygon(total_pts, pts, A, B, C, D)) {

/*************************************************************
 calculate the rotation about the y axis necessary to
 bring the normal into the y-z plane.
*************************************************************/

  angles_from_normal(A, B, C, 'y', &anglex, &angley);

/*************************************************************
 create the transformation matrix necessary to multiply
 with the array of vertices to generate the rotation.
*************************************************************/

  build_transform_matrix(transform1, angley, 'y', pts, pt1);

/*************************************************************
 perform the matrix multiplication with the array of
 vertices and store the new coordinates in the new array
 new_pts.
*************************************************************/

  map_pts(total_pts, pts, transform1, points,  new_pts);

/*************************************************************
 now extract three non-colinear vertices from the new
 array of vertices, new_pts.
*************************************************************/

  extract_3_pts(total_pts, new_pts, &pt1, &pt2, &pt3);

/*************************************************************
 since the polygon has been rotated about the y axis we
 calculate the coefficients for the equations to the
 polygon one more time.
*************************************************************/

  determine_coeffs(total_pts, new_pts, pt1, pt2, pt3, &A, &B, &C, &D);
/*************************************************************
 calculate the rotation about the x axis
**************************************************************/

   angles_from_normal(A, B, C, 'x', &anglex, &angley);

/*************************************************************
 create the transformation matrix
**************************************************************/

   build_transform_matrix(transform1, anglex, 'x', new_pts, pt1);

/*************************************************************
 perform the matrix multiplication
**************************************************************/

   map_pts(total_pts, new_pts, transform1, points,  new_pts);

/*************************************************************
 call the routine which will perform the decomposition
 upon the newly transformed polygon which now lies in the
 x-y-0 plane.(thus to be handled as a 2D polygon)
**************************************************************/

   split(total_pts, new_pts, pts);

/*************************************************************
 once the dynamically allocated memory for the array
 new-pts has served its purpose, the memory is freed.
**************************************************************/

   cfree(new_pts, 3*total_pts, sizeof(float));
   cfree(stack, total_pts, sizeof(Stack));

 }

/*************************************************************
 if not planar then the decomposition is not attempted.
**************************************************************/

  return;

}
```

```
/*****************************************************************
  ROUTINE: extract.c
******************************************************************/
  This routine searches the array of vertices to find three points which
  do not lie on the same line and returns the indices of these points
******************************************************************/

extract_3_pts(total_pts, pts, pt1, pt2. pt3)

int total_pts;
Coord pts[][3];
int *pt1, *pt2, *pt3;

{

  int count, finished;

  *pt1= 0; *pt2= 0; *pt3= 0; count= 0; finished= 0;

  /* The first vertex in the array (index=0) is selected for the first point. */

  while(count < total_pts && !finished) {

    /* We then search the rest of the array of vertices to locate two more */

    count+= 1;

    if(*pt2 == 0) {

      /* If the second point has not been selected yet then we check that
         x, y. and z coordinates, of the vertex currently under exam, are
         different. */

      if(pts[*pt1][0] != pts[count][0] ||
         pts[*pt1][1] != pts[count][1] ||
         pts[*pt1][2] != pts[count][2] )
         /* If so then the vertex becomes the second point. */
        *pt2= count; }

    else if(*pt3 == 0) {

      /* If the third point has not yet been selected, then we check that
         the coordinates of the vertex under examination are different
         from both the first and second point. */

      if((pts[*pt1][0] != pts[count][0] ||
          pts[*pt1][1] != pts[count][1] ||
         pts[*pt1][2] != pts[count][2] ) &&
        (pts[*pt2][0] != pts[count][0] ||
         pts[*pt2][1] != pts[count][1] ||
         pts[*pt2][2] != pts[count][2] ))
        /* If the coordinates are different then the vertex is selected
           as the third point. */
        *pt3= count; }

  else
     finished= 1;

  }

  return;

}
```

```
/*****************************************************************
  ROUTINE: is_planar.c
*****************************************************************/
  This routine takes each vertex of the polygon and uses it to solve the
  equation for the plane as calculated earlier.  If every vertex lies
  in the plane, then a value of 1 is returned.
*****************************************************************/
planar_polygon(total_pts, pts, A, B, C, D)

int total_pts;
Coord pts[][3];
float A, B, C, D;

{

  int count, planar;

  count= 0; planar= 1;

  while(count < total_pts && planar) {

    /* The equation for the plane is setup to equal zero if the coordinates
      of the vertex lies in the plane.  A range of .02 on either size of
      zero is allowed to handle round off error. This test is done for
      each vertex in the polygon.  If any vertex fails the test then the
      function returns (0) for non-planar. */

    if((A*(pts[count][0])+ B*(pts[count][1])+ C*(pts[count][2])- D) > .02 ||
      (A*(pts[count][0])+ B*(pts[count][1])+ C*(pts[count][2])- D) < -.02)
     planar= 0;

    count+= 1;


  }

  return(planar);

}
```

```
/*******************************************************************
 ROUTINE: coeffs.c
*******************************************************************/
 This routine uses the three vertices selected from the polygon to
 calculate the coefficients that will be used in both the equation
 of the plane the polygon lies in and the normal to that plane.
*******************************************************************/

determine_coeffs(total_pts, pts, pt1, pt2, pt3, A, B, C, D)

int   total_pts;
Coord pts[][3];
int   pt1, pt2, pt3;
float *A, *B, *C, *D;

{

  float x0, x1, x2, y0, y1, y2, z0, z1, z2;;

  x0= pts[pt1][0];
  y0= pts[pt1][1];
  z0= pts[pt1][2];

  x1= pts[pt2][0];
  y1= pts[pt2][1];
  z1= pts[pt2][2];

  x2= pts[pt3][0];
  y2= pts[pt3][1];
  z2= pts[pt3][2];

  *A= (y0-y1)*(z2-z1) - (y2-y1)*(z0-z1);
  *B= (z0-z1)*(x2-x1) - (z2-z1)*(x0-x1);
  *C= (x0-x1)*(y2-y1) - (x2-x1)*(y0-y1);

  *D= (*A)*pts[pt1][0] + (*B)*pts[pt1][1] + (*C)*pts[pt1][2];

  return;

}
```

```
/**************************************************************
 ROUTINE: angles.c
***************************************************************
 This routine utilizes the coefficients, determined from the normal to
 the polygon, to determine the angles of rotation necessary about the x
 and y axes.  These rotations are necessary to bring the polygon into the
 x-y plane.  This routine is called the first time to calculate the
 rotation about the y axis. The second time it is called is to determine
 the x axis rotation.  The parameter "axis" is used to pass which angle
 is to be calculated.
**************************************************************/

angles_from_normal(A, B, C, axis, anglex, angley)

float A, B, C;
char  axis;
float *anglex, *angley;

{

  float pi, degx, degy, degz;

  pi= 3.1415926536;

  degx= acos(A/sqrt(A*A+B*B+C*C))*360/(2*pi);
  degy= acos(B/sqrt(A*A+B*B+C*C))*360/(2*pi);
  degz= acos(C/sqrt(A*A+B*B+C*C))*360/(2*pi);

  if(A==0 && B==0) {
    /* simply translate the polygon to the origin */
    *anglex= 0.0;
    *angley= 0.0; }

  else if(A==0 && C==0) {
    /* rotate polygon 90 degrees about x axis */
    *anglex= 90.0;
    *angley=  0.0; }

  else if(B==0 && C==0) {
    /* rotate polygon 90 degrees about y axis */
    *anglex=  0.0;
    *angley= 90.0; }

  else {
    /* must calculate amount of rotation about the x & y axes */
    *anglex= acos(C/sqrt(B*B+C*C))*360/(2*pi);
    *angley= acos(C/sqrt(A*A+C*C))*360/(2*pi);
  if(axis == 'y') {
    if(degx < 90.0)
      *angley= -(*angley);
    *anglex= 0.0; }
  else {
    if(degy > 90.0) {
     *anglex= -(*anglex);
    *angley= 0.0; }
  }
}

/* convert the angles into tenths of degrees as needed by the IRIS system. */
*anglex= 10*(*anglex);
*angley= 10*(*angley);

return;

}
```

```
/*****************************************************************
 ROUTINE: transmatx.c
*****************************************************************/
 This routine takes a matrix structure and initializes it to an identity
 matrix. Then the necessary translation values and rotation values are
 added to the matrix to create the resultant transformation matrix.
*****************************************************************/

build_transform_matrix(transform1, angle, axis, pts, pt1)

Matrix transform1;
float  angle;
char   axis;
Coord  pts[][3];
int    pt1;

{
 int i, j;

 /* create the identity matrix */
 for(i=0; i<4; i+=1) {
  for(j=0; j<4; j+=1) {
   if(i==j)
    transform1[i][j]= 1;
   else
    transform1[i][j]= 0;
  }
 }
 pushmatrix();
 loadmatrix(transform1);

 /* add in the rotation values */
 rotate((int)angle, axis);

 /* add in the translation values */
 translate(-pts[pt1][0], -pts[pt1][1], -pts[pt1][2]);

 getmatrix(transform1);
 popmatrix();

 return;
}
```

```
/*****************************************************************
 ROUTINE: mapover.c
*****************************************************************/
 This routine places up to 4 vertices into a matrix at one time then
 performs a matrix multiplication with the transformation matrix to
 produce x, y, and z coordinates for the polygon that is translated
 and rotated some amount about the x and y axes. These new coordinates
 are restored in the array new_pts for further manipulation.
*****************************************************************/

map_pts(total_pts, pts, transform1, points, new_pts)

int    total_pts;
Coord  pts[][3];
Matrix transform1, points;
float  new_pts[][3];

{

  int count, count2, cnt;

  count= 0; count2= 0;

  while(count < total_pts) {

    cnt= 0;

    while(cnt < 4) {
      /* place the coordinates of a vertex into the matrix */
      points[cnt][0] = pts[count][0];
      points[cnt][1] = pts[count][1];
      points[cnt][2] = pts[count][2];
      points[cnt][3] = 1.0;

      if((count+= 1) < total_pts)
        cnt+= 1;
      else
        break;

    }

    /* save the state of the matrix stack and perform the multiplication */
    pushmatrix();

    loadmatrix(transform1);
    multmatrix(points);
    getmatrix(points);
    popmatrix();

    cnt= 0;

    while(cnt < 4) {
      /* move the newly calculated coordinates back into the vertices array */
      new_pts[count2][0]= points[cnt][0];
      new_pts[count2][1]= points[cnt][1];
      new_pts[count2][2]= points[cnt][2];

      if((count2+= 1) < count)
        cnt+= 1;
      else
        break;

    }

  }

  return;

}
```

```
/*************************************************************
 ROUTINE:              split.c
*************************************************************/
 This routine takes the array for the transformed polygon and calls
 the necessary functions and routines to decompose the polygon
 and pass the vertices of the convex polygons to the IRIS system
 for filling.
*************************************************************/


split(total_pts, pts, old_pts)

int   total_pts;
Coord pts[][3];
Coord old_pts[][3];

{

 Object  decompose_polygon();
 Object  whole_thing;
 boolean clockwise;
 point   *p1, *p2;

/*************************************************************
 Create the doubly linked circular list, "main chain", for the
 concave polygon represented by the pts array.  Also initialize
 flags and ptrs that will be used in decomposing the polygon.
*************************************************************/

 clockwise= clockwise_ordering(total_pts, pts);

 create_main_chain(clockwise, total_pts, pts);

/*************************************************************
 Decompose the main polygon and build the set of simple convex
 polygons.
*************************************************************/

 decompose_polygon(total_pts, pts, old_pts);

 return;

}
```

```
/*****************************************************************
 ROUTINE:               main_chain.c
*****************************************************************/
 Create the doubly linked circular list, "main chain", for the
 concave polygon represented by the pts array.  Also initialize
 flags and ptrs that will be used in decomposing the polygon.
*****************************************************************/

create_main_chain(clockwise, total_pts, pts)

boolean clockwise;
int     total_pts;
Coord   pts[][3];

{

 int count1;
 point *p1, *p2;

 /* allocate memory for a structure to contain data on the first
    vertes of the polygon */

 p1= new(point);

 /* set pointers to this structure and initialize all values */

 start_ptr= p1;
 current_ptr= p1;
 p1->index= 0;
 p1->direction= 0;
 p1->failed= FALSE;
 p1->segment_used= FALSE;

 /* if polygon vertices are ordered clockwise then we index the vertices
    array beginning at 0 and incrementing from there.  Other we must begin
    at the other end of the vertices array to extract our coordinates.  In
    this way a forward move along our doubly linked list will equal a
    clockwise move around the polygon. */

 if(clockwise)
  count1= 1;
 else
  count1= total_pts- 1;

 /* for each vertex of the polygon we build a point structure, calculate
    the slope, intercept, initilize flags and connect the structure
    via pointers to its preceeding and succeeding vertices in the
    polygon. The result is a doubly linked list of structures. */

  do{
    p2= new(point);
    p1->forward= p2;
    p2->back= p1;
    p1= p2;
    p1->index= count1;
    determine_slope(pts[p1->back->index][0], pts[p1->back->index][1],
              pts[p1->index][0], pts[p1->index][1], &(p1->slope),
              &(p1->intercept), &(p1->direction));
    p1->failed= FALSE;
    p1->segment_used= FALSE;
    if(clockwise)
      count1+= 1;
    else
      count1-= 1;
  }while(count1< total_pts && count1 > 0);

  /* connect the first vertex structure to the last vertex of the polygon
    and calculate the slope and intercept of the line segment the two
    create. */

  p1->forward= start_ptr;
  start_ptr->back= p1;
  p1= start_ptr;
  determine_slope(pts[p1->back->index][0], pts[p1->back->index][1],
              pts[p1->index][0], pts[p1->index][1], &(p1->slope),
            &(p1->intercept), &(p1->direction));

  return;

}
```

```
/*****************************************************************
 ROUTINE:              clockwis.c
*****************************************************************/
 This routine calculates the area of the arbitrary planar
 polygon once it has been transformed to the 2D x-y plane.
 A negative area indicates a clockwise ordering of the vertices
 while a positive area indicatesa a negative orderig of vertices
 A "0" is returned for counterclockwise ordering while a "1" is
 returned for clockwise ordering.
*****************************************************************/

clockwise_ordering(total_pts, pts)

int   total_pts;
Coord pts[][3];

{

  float area;
  int   i, result;

  area= 0.0;

  for(i=0; i< total_pts- 1; i+= 1) {
    area= area + pts[i][0] * pts[i+1][1];
  }

  area= area + pts[total_pts- 1][0] * pts[0][1];

  for(i=0; i< total_pts- 1; i+= 1) {
    area= area - pts[i][1] * pts[i+1][0];
  }

  area= area - pts[total_pts- 1][1] * pts[0][0];

  if(area >= 0)
    result= 0;
  else
    result= 1;

  return (result);

}
```

```
/**************************************************************
 ROUTINE:            decompose.c
**************************************************************/
 Decompose the main polygon and as  convex polygons are determined
 these are passed through calls to the IRIS system to be filled.
**************************************************************/

Object decompose_polygon(total_pts, pts, old_pts)

int   total_pts;
Coord pts[][3];
Coord old_pts[][3];

{

 boolean no_point_found, search_incomplete, adjust_needed, OK;
 boolean skip_flag, skip_search;
 int     pt1, pt2, poly_count, value;
 point   *working_ptr;

 clear_stack();
 get_pt(total_pts, pts);
 get_pt(total_pts, pts);

 search_incomplete= TRUE;
 skip_search= TRUE;
 poly_count= 0;

/**************************************************************
 Once an initial two points are taken from the concave polygon,
 the process moves through the main chain to select out points
 for constructing the simple convex polygons. Calls are made to
 various routines to check for validity of pts that are accepted
 into a polygon.
**************************************************************/

 while(search_incomplete) {
  adjust_needed= TRUE;
  OK= TRUE;
  while(OK) {
   get_pt(total_pts, pts);

   /* we first check to see if we have made a complete loop of the
     vertices. */

   if(loop_completed()) {
    working_ptr= start_ptr;
 /* find the structure in the linked list that matches the
    vertex on top of the stack */

 while(working_ptr->index != stack[TOP].index) {
  working_ptr= working_ptr->forward;
 }

 /* check to see if vertex creates a line segment with the
    vertex below it, on the stack, that intersects any edges
    of the concave polygon */

 if(stack[TOP-1].index != working_ptr->back->index) {
  value= intersecting_lines(pts[stack[TOP].index][0],
        pts[stack[TOP].index][1], pts[stack[TOP-1].index][0],
       pts[stack[TOP-1].index][1], stack[TOP].slope,
       stack[TOP].intercept, total_pts, pts);
 }

 if(value) {

   /* if vertex creates an intersecting line segment then we
      remove it from the stack and reset the pointer to the
      linked list */

  current_ptr= start_ptr;
 while(current_ptr->index != stack[TOP-1].index) {
  current_ptr= current_ptr->forward;
  }

 current_ptr= current_ptr->forward;
 release_pt();
  release_pt(); }

 else {

 /* since the line segment create is OK, we simply remove the
    vertex from the stack since it is the same as the one on
    the bottom of the stack, namely the beginning vertex */

  OK= FALSE;
  release_pt();
}
}

/* if we have not completed the loop then we must conduct the
  tests to ensure valid selection of vertices for a convex
```

```
    polygon */

  else if(pt_out_of_bounds(total_pts, pts)) {
   release_pt();
  }
}

/* if at least three points have been selected then... */

if(valid_pts()) {

  /* save the beginning and ending vertices for future reference */

  pt1= stack[BOTTOM].index;
  pt2= stack[TOP].index:

  /* pass the vertices to IRIS system hardware for filling and display */

  define_polygon(old_pts);
  poly_count-= 1:

  /* if this is a normal decomposition and not a "fill in" polygon to
    cover holes left in the concave polygon the the do the following */

  if(skip_search) {

    /* set the flags that indicate next time through this loop we
      will be working on a "fill in" polygon */

    adjust_needed= FALSE;
    skip_search= FALSE;

    /* adjust the pointers to the linked list so next time through
      the loop we have the start and end vertices of the previous
      polygon to begin working with */

    working_ptr= start_ptr;
    while(working_ptr->index != pt2) {
     working_ptr= working_ptr->forward;
    }

    /* if the start and end vertices of the previous polygon are
      adjacent on the concave polygon then forget about building
      a "fill in" polygon */

    if(working_ptr->forward->index != pt1) {
     while(current_ptr->index != pt1) {
          current_ptr= current_ptr->forward:
          }
          get_pt(total_pts, pts);
          current_ptr= working_ptr;
        get_pt(total_pts, pts); }
        else {
        adjust_needed= TRUE;
        }
      }
    }

    /* if the polygon is not fully decomposed yet we select
      two new vertices to continue the decomposition process */

    if(adjust_needed) {
      skip_search= TRUE;
      if(object_complete(total_pts))
        search_incomplete= FALSE;
      else {
        adjust_search_area(pts);
        clear_stack();
        get_pt(total_pts, pts);
        get_pt(total_pts, pts);
      }
    }
  }

  return;

}
```

```
/************************************************************
 ROUTINE:              clear_stack.c
************************************************************/
 This routine simply resets the stack pointer to -1. In this
 manner when the stack has its first item pushed on, the pointer
 will accurately reflect a value of 0. (BOTTOM OF STACK)
************************************************************/

clear_stack()

{

  TOP= -1;

}
```

```
/*****************************************************************
 ROUTINE:              get_pt.c
*****************************************************************/
 This retrieves the next point in the main chain, as identified
 by the current_ptr, and places the point on the top of the stack
 In doing so it increments the TOP variable.  Also as the point
 is retrieved, the slope, intercept, and direction of the line
 segment it creates with the vertex below it on the stack is
 calculated.
*****************************************************************/

get_pt(total_pts, pts)

int   total_pts;
Coord pts[][3];

{
 TOP+= 1;

 if(TOP <= total_pts) {

   /* select the vertex in the linked list that is currently pointed to */
   stack[TOP].index= current_ptr->index;
   /* and then advance the list pointer */
   current_ptr= current_ptr->forward;
   if(TOP > 0) {
     /* take the x, y, and z coordinates for both the vertex on top of
        the stack and the vertex just below it, to calculate the slope
        and y-intercept of the line segment they create */
     determine_slope(pts[stack[TOP-1].index][0], pts[stack[TOP-1].index][1],
                pts[stack[TOP].index][0], pts[stack[TOP].index][1],
                &stack[TOP].slope, &stack[TOP].intercept,
                &stack[TOP].direction);
   }
 }
 else {
   printf("STACK overflow occured while retrieving another point");
 }

 return;

}
```

```
/*****************************************************************
 ROUTINE:            release_pt.c
*****************************************************************/

 This routine removes the item on top of the stack by
 decrementing the top of stack pointer TOP.
*****************************************************************/

release_pt()
{

  if(TOP >= BOTTOM)
    TOP-= 1;
  else {
    printf("STACK underflow occured while trying to release a non-");
    printf("existence point.");
  }

  return;

}
```

```
/****************************************************************
 ROUTINE:            slopes.c
****************************************************************/
 This works through the pts array. and the stack array. By
 taking value of the TOP of stack and the point beneath it.
 the corresponding coordinates are obtained from the pts array
 and a slope, intercept and direction are calculated. These
 values are then stored in the stack.
****************************************************************/

determine_slope(x1, y1. x2, y2. slope. intercept, direction)

Coord x1, y1. x2, y2:
float *slope:
Coord *intercept:
int   *direction:

{

 Coord dx, dy:

 /* calculate the delta x and delta y for the to points */

 dx= x2- x1;
 dy= y2- y1;

 /* if the line segment is vertical then use the value 99999.0 to
   indicate such (avoiding division by zero problems) */

 if(dx == 0) {
  *slope= 99999.0;
  *intercept= x2; }
 else {
  *slope= dy/dx;
  *intercept= y2 - (dy/dx) * x2:
 }

 /* based upon the changes in delta x and delta y we determine if we are
   traversing down a slope or up a slope */

 if(dy < 0) {
  if(dx < 0)
   *direction= 1;
  else
   *direction= 0;
 }
```

```
  else {
     if(dx > 0)
       *direction= 0;
     else
       *direction= 1;
  }

  return;

}
```

```
/*******************************************************************
 ROUTINE:            compare.c
********************************************************************
 This routine determines if a point lies to the left or right of
 a line segment. This is done by using the x and y coordinates
 of the point with the slope, intercept, and direction of the
 line segment to create a parallel line segment. The y intercept
 (or in the case of vertical line, the x intercept) is used in
 conjunction with the direction of traversal along the line
 segment to find which side of the line segment, the point lies
 on.
*******************************************************************/

compare_boundry(x1,y1,slope,intercept,direction)

Coord x1, y1;
float slope;
Coord intercept;
int   direction;

{

 float test_value;
 int   value;

 /* if the line segment is vertical then we use the x-intercept to
    determine if a vertex lies to the left or right of another line
    segment */

 if(slope == 99999.0)
  test_value= x1;
 else
  test_value= y1- slope * x1;

 /* based upon traversal up or down a sloping line we determine if
    a vertex is to the left or right of the line segment */

 switch (direction) {

  case 0: if(test_value > intercept)
            value= 1;
          else
           value= 0;
          break;

  case 1: if(test_value < intercept)
            value= 1;
          else
            value= 0;
            break;
    }

    return (value);

  }
```

```
/*************************************************************
 ROUTINE:              pt_OOB.c
*************************************************************
 This routine takes the vertex index from the top of the stack
 and uses its x, y coordinates to determine if the point lies
 out of bounds with respect to the convex polygon being
 created.
*************************************************************/

pt_out_of_bounds(total_pts, pts)

int   total_pts;
Coord pts[][3];

{
 Coord   x1, y1;
 int     ptr, value;
 boolean not_finished;
 point   *working_ptr;

 /* obtain the coordinates for the vertex on top of the stack */

 x1   = pts[stack[TOP].index][0];
 y1   = pts[stack[TOP].index][1];
 ptr  = TOP- 1;
 value= 0;

 /* use the above coordinates to determine if the vertex lies to
    the left (out of bounds) with respect to any line segment
    created thus far in the selection of vertices */

 while (ptr > BOTTOM && !value) {
  value= compare_boundry(x1,y1,stack[ptr].slope,stack[ptr].intercept,
       stack[ptr].direction);
  ptr-= 1;
 }

/*************************************************************
 If point is not out of bounds with any line segments created
 then we check to see if the segment produced by this new point
 addition causes the start point for the polygon to fall out of
 bounds.
*************************************************************/

 if(!value) {
  x1= pts[stack[BOTTOM].index][0];
  y1= pts[stack[BOTTOM].index][1];
```

```
    value= compare_boundry(x1,y1,stack[TOP].slope,stack[TOP].intercept,
        stack[TOP].direction);
    if(value) {
      release_pt();
      current_ptr= current_ptr->back;
      working_ptr= start_ptr;
    }
  }

/***************************************************************
  If the newly created line segment does not cause the initial pt
  of the developing polygon to fall out of bounds, then we check
  to see if the line segment intersects any line segments of the
  concave polygon.
***************************************************************/

  if(!value) {
    working_ptr= start_ptr;
    while(working_ptr->index != stack[TOP].index) {
      working_ptr= working_ptr->forward;
    }
    if(stack[TOP-1].index != working_ptr->back->index) {

       /* we use the coordinates for the vertex at the top of stack
          and the vertex below it to create a line segment which we
          then check for intersection with any of the edges of the
          original concave polygon */

       value= intersecting_lines(pts[stack[TOP].index][0],
             pts[stack[TOP].index][1], pts[stack[TOP-1].index][0],
           pts[stack[TOP-1].index][1], stack[TOP].slope,
           stack[TOP].intercept, total_pts, pts);
    }
  }

/***************************************************************
  If the newly created line segment does not intersect any line
  segments of the concave polygon then we check to see if the
  line segment of the original polygon, that the top of stack
  point heads, causes the point below top of stack to fall out.
***************************************************************/

  if(!value) {
    if(current_ptr->back->back->index != stack[TOP-1].index) {
      x1= pts[stack[TOP-1].index][0];
      y1= pts[stack[TOP-1].index][1];
      value= compare_boundry(x1,y1,current_ptr->back->slope,
                            current_ptr->back->intercept,
                             current_ptr->back->direction);
      }
    }

/**********************************************************************
  If the point is determined to be out of bounds, then the
  appropriate pointers are adjusted to continue the decomposition
  process. Otherwise the pt is a valid addition to the stack
**********************************************************************/

  if(value) {
    working_ptr= start_ptr;
    not_finished= TRUE;
    do {
      if(working_ptr->index == stack[TOP].index) {
        not_finished= FALSE;
       if(working_ptr->failed == FALSE)
         working_ptr->failed= TRUE;}
      else {
        working_ptr= working_ptr->forward;
      }
    } while(not_finished);
  }

  return(value);

}
```

```
/****************************************************************
 ROUTINE:          loop_complet.c
*****************************************************************
 This routine checks to see if the vertices of the concave
 polygon have been completely traversed while developing a single
 convex polygon.  This is done when the vertex on top of the
 stack is the same vertex on the bottom of the stack. When this
 occurs a value of 1 is returned.
****************************************************************/

int loop_completed()
{

  int value;

  if(TOP> -1) {
    if(stack[TOP].index == stack[BOTTOM].index) {
      value= 1; }
    else {
      value= 0;
    }
  }
  else {
    printf("Error in stack index while checking TOP & BOTTOM values");
    value= -1;
  }

  return (value);

}
```

```
/*****************************************************************
 ROUTINE:             adjust.c
******************************************************************
 This routine searches the circular list of vertices, checking
 the various condition flags, to locate a vertex or line segment
 of the originial polygon which has not been utilized.  As one is
 located, the list's currency pointer is moved to that structure
 in the list so that further polygon decomposition may continue
*****************************************************************/

adjust_search_area(pts)

Coord pts[][3];

{

 boolean done;
 point   *p1;

 done= 0;
 p1= start_ptr;

 do {
   if(p1-> failed == 1)
     /* check to see if the line segment to either side of the vertex
        has been used in a previously developed polygon. */
     if(!(p1-> forward-> segment_used) || !(p1-> segment_used)) {
       current_ptr= p1;
       p1-> failed= 0;
       done= 1; }
     else {
       p1= p1-> forward;
     }
   else {
     p1= p1-> forward;
   }
 } while(!done && p1 != start_ptr);

 return;

}
```

```
/*****************************************************************
  ROUTINE:              valid_pts.c
******************************************************************
  This routine checks to ensure that at least 3 vertices have
  been determined for the convex polygon.  If not then the
  value of 0 is returned.
*****************************************************************/

boolean valid_pts()
{

  int value;

  if(TOP >= 2)
    value= 1;
  else
    value= 0;

  return (value);

}
```

```
/*****************************************************************
 ROUTINE:            define_poly.c
******************************************************************
 This routine takes the vertices accumulated on the stack and
 uses the IRIS features pmv and pdr to actually perform the
 display of the polygon created.  The index for each vertex
 is used to access the original array of points to get the x, y
 and z coordinates used.
*****************************************************************/

define_polygon(pts)

Coord  pts[][3];

{

 int   count;
 point *track_ptr;

 track_ptr= start_ptr;

 /* pass the starting vertex for the newly created convex
   polygon to the IRIS hardware with a point move command */

 pmv(pts[stack[BOTTOM].index][0],
    pts[stack[BOTTOM].index][1],
    pts[stack[BOTTOM].index][2]);

 /* locate this vertex in the linked list */

 while(track_ptr->index != stack[BOTTOM].index) {
   track_ptr= track_ptr->forward;
 }

 /* set the flag to indicate the vertex has been used
   in a convex polygon */

 if(track_ptr->back->index == stack[TOP].index)
   track_ptr->segment_used= TRUE;

 /* for each vertex of the convex polygon, pass its
   coordinates to IRIS hardware with the point draw
   command */

 count= 1;
 while(count <= TOP) {
  while(track_ptr->index != stack[count].index) {
        track_ptr= track_ptr->forward;
      }
      if(track_ptr->back->index == stack[count- 1].index)
        track_ptr->segment_used= TRUE;
      pdr(pts[stack[count].index][0],
         pts[stack[count].index][1],
         pts[stack[count].index][2]);
      count+= 1;
    }

    /* pass the command to the IRIS hardware that the
       fill the completed convex polygon */

    pclos();

   clear_stack();

    return;

  }
```

```
/*****************************************************************
 ROUTINE:            obj_complet.c
*****************************************************************
 This routine checks each vertex and edge of the original
 concave polygon to see if they have all been utilized in the
 process of decomposition. If so a value of 1 is returned.
*****************************************************************/

int object_complete(total_pts)

int total_pts;

{

  point  *p1;
  int    count, value;

  count= 0;
  value= 1;
  p1= start_ptr;

  /* search through the linked list of structures, for the
     vertices, and return the index of the first vertex that
     creates an edge of the concave polygon not yet utilized */
  do {
    if(p1-> segment_used) {
       p1= p1-> forward;
       count += 1; }
    else {
       value= 0;
    }
  } while(count < total_pts && value == 1);

  return (value);

}
```

```
/*****************************************************************
 ROUTINE:          intersecting.c
*****************************************************************
 This routine takes two vertices and the slope and intercept
 of the line segment they create to determine if it intersects
 any of the edges of the original polygon. If so then a value
 of 1 is returned.
*****************************************************************

intersecting_lines(x2, y2, x1, y1, slope, intercept, total_pts, pts)

Coord x2, y2, x1, y1;
float slope;
Coord intercept;
int   total_pts;
Coord pts[][3];

{

 point *p1;
 float x_intersect, y_intersect, x_min, x_max, y_min, y_max, fudge;
 int   value;

 /* define a value to offset round off error */

 fudge= 0.000001;

 value= 0;
 p1   = start_ptr;

 /* for each edge of the concave polygon we check if either vertex
    lies on the edge. If not then we determine the intersection
    of the line, defined by the edge, with the line defined by the
    two vertices. Finally we determine if this intersection lies on
    both line segments.  If so then we have found an actual intersection. */

 do {

  /* check if first vertex lies on the polygon's edge being examined */

  if(((x1 != pts[p1->index][0]) || (y1 != pts[p1->index][1])) &&
    ((x1 != pts[p1->back->index][0]) || (y1 != pts[p1->back->index][1]))) {

   /* check if second vertex lies on the edge */

   if(((x2 != pts[p1->index][0]) || (y2 != pts[p1->index][1])) &&
     ((x2 != pts[p1->back->index][0]) || (y2 != pts[p1->back->index][1]))) {
```

```
/* if the lines are not parallel then ... */

if(slope != p1->slope) {

  /* if the line created by the vertices is vertical */

  if(slope == 99999.0) {
    /* determine the coordinates of the intersection */
   x_intersect= x2;
   y_intersect= p1->slope * x_intersect+ p1->intercept; }

  /* if the edge of the polygon is vertical */

  else if(p1->slope == 99999.0) {
    /* determine the coordinates of the intersection */
   x_intersect= pts[p1->index][0];
   y_intersect= slope * x_intersect+ intercept; }

  /* if neither line is vertical */

  else {
    /* determine the coordinates of the intersection */
    x_intersect= (intercept - (p1->intercept))/((p1->slope) - slope);
   y_intersect= slope * x_intersect+ intercept;
  }

  /* determine the x range that the intersection must occur in
     to lie on the line segment created by the two vertices */

 max_min(x2,x1,&x_max,&x_min);

  /* check if the intersection falls within this range */

 if((x_intersect >= x_min-fudge)&&(x_intersect <= x_max+fudge)) {

    /* since the intersection is within this x range we now determine
       the x range for the edge of the concave polygon */

   max_min(pts[p1->index][0], pts[p1->back->index][0], &x_max, &x_min);

    /* check if the intersection falls within this range */

    if((x_intersect >= x_min-fudge)&&(x_intersect <= x_max+fudge)) {

      /* now that the intersection falls within the x range of both
         line segments, we must calculate the y range for each
```

```
              segment and perform the checks */

            /* y range for the vertices line segment */

             max_min(y2,y1,&y_max,&y_min);

            if((y_intersect >= y_min-fudge)&&(y_intersect <= y_max+fudge)) {

              /* y range for the concave polygon edge */

              max_min(pts[p1->index][1],pts[p1->back->index][1],&y_max,& y_min);
              if((y_intersect >= y_min-fudge)&&(y_intersect <= y_max+fudge)) {
                value= 1;
                /* the intersection exists for the actual line segments and
                  therefore the vertex on top of the stack must be removed */
              }
             }
            }
           }
          }
         }
        }
        p1= p1->forward;
      } while((p1 != start_ptr)&&(value == 0));

      return(value);

    }
```

```
/*********************************************************************
 ROUTINE:              max_min.c
**********************************************************************
 Given two float values, p1 and p2, this routine determines
 which is larger of the two.
**********************************************************************/

max_min(p1, p2, max, min)

Coord p1, p2, *max, *min;

{

 if(p1 < p2) {
   *max= p2;
   *min= p1; }
 else {
   *max= p1;
   *min= p2;
 }

 return;

}
```

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Thesis
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